Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0238112211

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2023**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (CE/CT) 325168

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Solve the inequality $3x^2 - 12x + 16 > 3x + 4$. [3]

(b) **(i)** Write $3x^2 - 12x + 16$ in the form $a(x+b)^2 + c$ where a, b and c are integers. [3]

(ii) Hence, write down the equation of the tangent to the curve $y = 3x^2 - 12x + 16$ at the minimum point of the curve. [1]

2 A curve has equation $y = 32x^2 + \frac{1}{8x^2}$ where $x \neq 0$.

(a) Find the coordinates of the stationary points of the curve. [5]

(b) These stationary points have the same nature. Use the second derivative test to determine whether they are maximum points or minimum points. [3]

3 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Show that $x+3$ is a factor of $-12+23x+3x^2-2x^3$. [1]

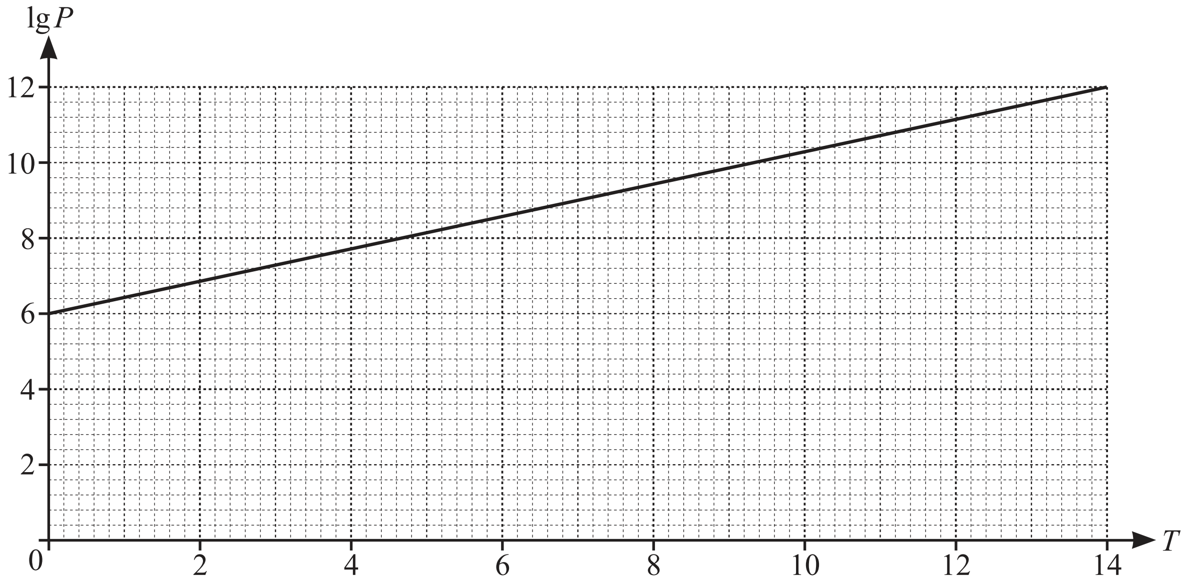
(b) The curve $y=-5+33x+3x^2-2x^3$ and the line $y=10x+7$ intersect at three points, A, B and C. These points are such that the x-coordinate of A has the least value and the x-coordinate of C has the greatest value. Show that B is the mid-point of AC. [7]

4 Variables x and y are related by the equation $y = 2 + \tan(1 - x)$ where $0 \leqslant x \leqslant \frac{\pi}{2}$. Given that x is increasing at a constant rate of 0.04 radians per second, find the corresponding rate of change of y when $y = 3$. [6]

5 Variables P and T are known to be connected by the relationship $P = Ab^T$, where A and b are constants. Values of P are found for certain values of time, T.

(a) Show that a graph of $\lg P$ against T will be a straight line. [2]

(b)



The diagram shows the graph of $\lg P$ against T. The graph passes through (0, 6) and (14, 12). Find the values of A and b. [4]

(c) Using the graph or otherwise, find the length of time for which P is between 100 million and 1000 million. [3]

6 **(a)** **(i)** Find the first three terms in the expansion of $\left(1+\frac{x}{7}\right)^5$, in ascending powers of x. Simplify the coefficient of each term. [2]

(ii) The expansion of $7(1+x)^n\left(1+\frac{x}{7}\right)^5$, where n is a positive integer, is written in ascending powers of x. The first two terms in the expansion are $7+89x$. Find the value of n. [2]

(b) In the expansion of $(k-2x)^8$, where k is a constant, the coefficient of x^4 divided by the coefficient of x^2 is $\frac{5}{8}$. The coefficient of x is positive. Form an equation and hence find the value of k. [5]

7 **(a)** $f(x) = \sqrt{3+(4x-2)^5}$ where $x > 1$.

Find an expression for $f'(x)$, giving your answer as a simplified algebraic fraction. [3]

(b) Variables x and y are related by the equation $y = \dfrac{5x}{3x+2}$. Using differentiation, find the approximate change in x when y increases from 10 by the small amount 0.01. [4]

(c) **(i)** Differentiate $y = x^3 \ln x$ with respect to x. [2]

(ii) Hence find $\int \left(\frac{x^2}{6}(2 + 3\ln x) \right) \mathrm{d}x$. [3]

8 A curve has equation $y = \cos\frac{x}{4}$ where x is in radians. The normal to the curve at the point where $x = \frac{4\pi}{3}$ cuts the x-axis at the point P. Find the exact coordinates of P. [7]

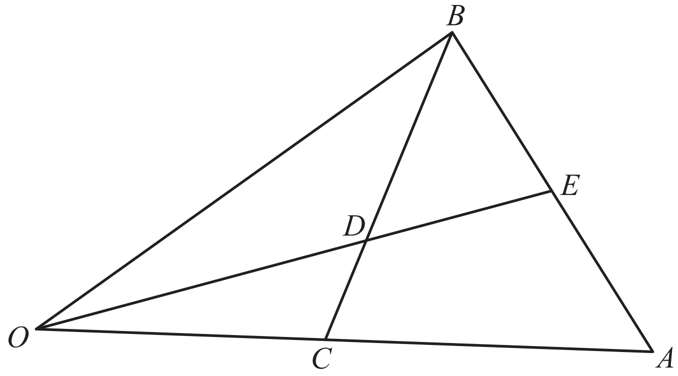
9 A particle travels in a straight line so that, t seconds after passing a fixed point, its velocity, $v\,\text{ms}^{-1}$, is given by

$$v = e^{\frac{t}{4}} \qquad \text{for } 0 \leqslant t \leqslant 4,$$

$$v = \frac{16e}{t^2} \qquad \text{for } 4 \leqslant t \leqslant k.$$

The total distance travelled by the particle between $t = 0$ and $t = k$ is 13.4 metres. Find the value of k. [6]

10



The diagram shows a triangle *OAB*. The point *C* is the mid-point of *OA*. The point *D* lies on *CB* such that $CD : DB = 2 : 3$.

$$\overrightarrow{OC} = \mathbf{c} \qquad \overrightarrow{CB} = \mathbf{b}$$

The point *E* lies on *AB* such that $\overrightarrow{OE} = \lambda\overrightarrow{OD}$ and $\overrightarrow{AE} = \mu\overrightarrow{AB}$ where λ and μ are scalars.
Find two expressions for $\overrightarrow{OE}$, each in terms of **b**, **c** and a scalar, and hence find $AE : EB$. [8]

Continuation of working space for Question 10.

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